Homework Set #1 Solution
Thermodynamics II
Due: 1-30-97

Homework 1, Problem 1:

The airlock to the containment of a nuclear power station is at 14.7 psia, 70°F. The airlock is insulated. Over some period of time, the air lock is pressurized to 12 psig from an air header that is at 50 psig, 50°F. Calculate the final temperature in the air lock. The air lock volume is 80 ft^3. Solve first using constant specific heats and then again using temperature dependent specific heats. Compare the two solutions. When solving using constant specific heats, formulate your mathematical model so that you always end up with differences in internal energies or enthalpies. DO NOT set $h=c_p*T$. This is not correct, and you will receive NO CREDIT FOR THE PROBLEM!

Known quantities:

$P_1 := 14.7 \cdot psi$

$T_1 := (77 + 460) \cdot R$

$V_1 := 80 \cdot ft^3$

$P_2 := 22 \cdot psi$

$V_2 := V_1$

$R_a := \frac{8314}{28.97} \cdot \frac{joule}{kg \cdot R}$

$T_{in} := (50 + 460) \cdot R$

$P_{in} := (50 + 14.7) \cdot psi$

$c_p := 1000 \cdot \frac{joule}{kg \cdot K}$

Mass conservation:

$$m_{in} = m_2 - m_1$$

First law:

$$Q - W_a + (H_{in} - H_{out}) = U_2 - U_1 \qquad H_{in} = m_{in} \cdot h_{in}$$

Implementing the first law:

$$U_2 = H_{in} + U_1 = (m_2 - m_1) \cdot h_{in} + m_1 \cdot u_1$$

$$u_2 = \frac{(m_2 - m_1) \cdot h_{in} + m_1 \cdot u_1}{m_2}$$

Solution using the tables:

$$u_2 = \frac{(m_2 - m_1)\cdot h_{in} + m_1 \cdot u_1}{m_2}$$

$$m_2(T) := \frac{P_2 \cdot V_2}{R_a \cdot T} \qquad m_1 := \frac{P_1 \cdot V_1}{R_a \cdot T_1}$$

Tabular data for air:

$$u_1 := 92.04 \cdot \frac{BTU}{lb} \qquad h_{in} := 119.48 \cdot \frac{BTU}{lb}$$

$$m_1 = 3.285 \cdot lb$$

Define an error function for iteration:

$$F(T_2, u_2) := u_2 - \frac{\left(\frac{P_2 \cdot V_2}{R_a \cdot T_2} - \frac{P_1 \cdot V_1}{R_a \cdot T_1}\right)\cdot h_{in} + m_1 \cdot u_1}{\frac{P_2 \cdot V_2}{R_a \cdot T_2}}$$

$$F_1 := F\left(T_{21}, 109.21 \cdot \frac{BTU}{lb}\right) \qquad F_1 = 11.582 \cdot \frac{BTU}{lb} \qquad u_{21} := 109.21 \cdot \frac{BTU}{lb} \quad T_{21} := 640 \cdot R$$

$$F_2 := F(T_{22}, u_{22}) \qquad F_2 = -9.003 \cdot \frac{BTU}{lb} \qquad u_{22} := 92.04 \cdot \frac{BTU}{lb} \quad T_{22} := 540 \cdot R$$

$$T_{23} := T_{21} - \left(\frac{T_{22} - T_{21}}{F_2 - F_1}\right)\cdot F_1 \qquad T_{23} = 583.736 \cdot R$$

Use the temperature value that is closest to the interpolated value:

$$T_{23} := 580 \cdot R$$

$$u_{23} := 98.9 \cdot \frac{BTU}{lb}$$

$$F_3 := F(T_{23}, u_{23}) \qquad F_3 = -0.777 \cdot \frac{BTU}{lb}$$

$$T_{24} := T_{22} - \left(\frac{T_{23} - T_{22}}{F_3 - F_2}\right)\cdot F_2 \qquad T_{24} = 583.778 \cdot R$$

Close enough

Solution using constant specific heats:

$$u = h - P\cdot v$$

$$h_2\cdot m_2 = m_2\cdot h_{in} - m_1\cdot h_{in} + m_1\cdot h_1 - P_1\cdot V_1 + P_2\cdot V_2$$

$$m_2\cdot\left(h_2 - h_{in}\right) = m_1\cdot\left(h_1 - h_{in}\right) + V_1\cdot\left(P_2 - P_1\right)$$

$$\frac{P_2\cdot V_2}{R_a\cdot T_2}\cdot c_p\cdot\left(T_2 - T_{in}\right) = m_1\cdot c_p\cdot\left(T_1 - T_{in}\right) + V_1\cdot\left(P_2 - P_1\right)$$

$$T_2 := \frac{P_2\cdot V_2\cdot c_p\cdot T_{in}}{P_2\cdot V_2\cdot c_p - R_a\cdot\left[m_1\cdot c_p\cdot\left(T_1 - T_{in}\right) + V_1\cdot\left(P_2 - P_1\right)\right]}$$

$$T_2 = 641.513\cdot R \qquad T_2 - 460\cdot R = 181.513\cdot R$$

Homework 1, Problem 2:

Determine the entropy generation for the process in problem 1. Explain where and why entropy is generated, and what it physically represents.

$$S_2 - S_1 = \sigma + m_{in}\cdot s_{in} = \sigma + m_2\cdot s_{in} - m_1\cdot s_{in}$$

$$\sigma = m_2\cdot\left(s_2 - s_{in}\right) + m_1\cdot\left(s_1 - s_{in}\right) = m_2\cdot\left(c_p\cdot\ln\left(\frac{T_2}{T_{in}}\right) - R_a\cdot\ln\left(\frac{P_2}{P_{in}}\right)\right) + m_1\cdot\left(c_p\cdot\ln\left(\frac{T_1}{T_{in}}\right) - R_a\cdot\ln\left(\frac{P_1}{P_{in}}\right)\right)$$

$$\sigma := m_2\left(T_2\right)\cdot\left(c_p\cdot\ln\left(\frac{T_2}{T_{in}}\right) - R_a\cdot\ln\left(\frac{P_2}{P_{in}}\right)\right) + m_1\cdot\left(c_p\cdot\ln\left(\frac{T_1}{T_{in}}\right) - R_a\cdot\ln\left(\frac{P_1}{P_{in}}\right)\right)$$

$$\sigma = 1.414\cdot\frac{BTU}{R}$$

Homework 1, Problem 3:

A small building is 4.267 m wide, 9.144 m long, and 3.048 ft high. Heat transfer through the floors, walls, and ceiling is proportional to the temperature difference across these surfaces, as shown in the following equation:

$$\dot{Q} = (UA)_{surface} \times \left(T_{exterior} - T_{inside}\right)$$

The variable U has units of W/m2-K, and depends on the material used to build the wall. These are directly related to the R-values you hear about. The U-values for the ceiling, floor, and walls are provided below. If the flow rate of air into the building from the outside is such that the building volume changes once per hour, and if there is a heater that provides 3 kW of energy to the building, calculate the building inside air temperature.The outside air temperature is -10°C, and the ground temperature under the floor is 5°C.

$U_{floor} := 0.5 \cdot \frac{watt}{K \cdot m^2}$ $U_{ceiling} := 0.13 \cdot \frac{watt}{K \cdot m^2}$ $U_{walls} := 029 \cdot \frac{watt}{K \cdot m^2}$

$l := 9.144 \cdot m$

$w := 4.267 \cdot m$ $Q_h := 100000 \cdot \frac{BTU}{hr}$ $Q_h = 2.931 \cdot 10^4 \cdot watt$

$h := 3.048 \cdot m$ $Q_{htr} := 70000 \cdot watt$ $T_{outside} := (-10 + 273) \cdot K$

$T_{ground} := (5 + 273) \cdot K$

$A_w := (2 \cdot l \cdot h) + 2 \cdot w \cdot h$ $A_{ceil} := l \cdot w$ $A_{floor} := l \cdot w$

$A_w = 81.753 \cdot m^2$ $A_{ceil} = 39.017 \cdot m^2$ $A_{floor} = 39.017 \cdot m^2$

Mass Flow rate (needs to be a function of the inside air temperature):

$\rho_{air}(T) := \frac{(101 \cdot kPa)}{R_a \cdot T}$ $T_{in} := 300 \cdot K$ $\rho_{air}(300 \cdot K) = 0.652 \cdot \frac{kg}{m^3}$

$ACH := \frac{1}{hr}$

$Vol := l \cdot w \cdot h$ $Vol = 118.925 \cdot m^3$

$md_{air}\left(T_{in}\right) := (ACH \cdot Vol) \cdot \rho_{air}\left(T_{in}\right)$

$md_{air}\left(T_{in}\right) = 77.50682 \cdot \frac{kg}{hr}$

$Q - W_a + H_{in} - H_{out} = 0$ $W_a = 0 \cdot watt$ $H_{in} = md_{air} \cdot h_{out}$ $H_{out} = md_{air} \cdot h_{room}$

$Q = Q_{htr} + Q_{walls} + Q_{floor} + Q_{ceil}$

$$Q_{htr} + Q_{walls} + Q_{floor} + Q_{ceil} + md_{air} \cdot \left(h_{outside} - h_{room}\right) = 0$$

$$Q_{walls}(T) := U_{walls} \cdot A_{w} \cdot \left(T_{outside} - T\right)$$

$$Q_{ceil}(T) := U_{ceiling} \cdot A_{ceil} \cdot \left(T_{outside} - T\right)$$

$$Q_{floor}(T) := U_{floor} \cdot A_{floor} \cdot \left(T_{ground} - T\right)$$

$$c_{p} := 1000 \cdot \frac{joule}{kg \cdot K}$$

$$F(T) := Q_{htr} + Q_{walls}(T) + Q_{floor}(T) + Q_{ceil}(T) + md_{air}(T) \cdot c_{p} \cdot \left(T_{outside} - T\right)$$

$$T := 300 \cdot K$$

$$T_{inside} := root(F(T), T)$$

$T_{inside} - 273 \cdot K = 19.076 \cdot K$ (This is actually in °C)